[2] $\mathcal{L}\{t\cos 5t\} = -\frac{d}{ds}\mathcal{L}\{\cos 5t\}$

$$= \boxed{-\frac{d}{ds}\frac{s}{s^2+25}} \quad \frac{1}{2}$$

$$= \boxed{-\frac{(1)(s^2+25)-s(2s)}{(s^2+25)^2}} \quad 1$$

$$= \boxed{\frac{s^2-25}{(s^2+25)^2}} \quad \frac{1}{2}$$

$\mathcal{L}\{te^{2t+3}\cos 5t\} = e^3\mathcal{L}\{e^{2t}\cdot t\cos 5t\}$

$$= \frac{1}{2}\ \boxed{e^3}\cdot\boxed{\frac{(s-2)^2-25}{((s-2)^2+25)^2}} \quad \frac{1}{2}$$

$$= \frac{e^3(s^2-4s-21)}{(s^2-4s+29)^2}$$

[3] $f(t) = \boxed{\begin{cases} 5-2t, & 0<t<2 \\ t-1, & t>2 \end{cases}}$ $\frac{1}{2}$

$$\mathcal{L}\{f(t)\} = \boxed{\int_0^2 e^{-st}(5-2t)\,dt + \int_2^\infty e^{-st}(t-1)\,dt} \quad \frac{1}{2}$$

$$\boxed{\begin{array}{cc|cc} \underline{u} & \underline{dv} & \underline{u} & \underline{dv} \\ 5-2t & e^{-st} & t-1 & e^{-st} \\ -2 & -\frac{1}{s}e^{-st} & 1 & -\frac{1}{s}e^{-st} \\ 0 & \frac{1}{s^2}e^{-st} & 0 & \frac{1}{s^2}e^{-st} \end{array}} \quad 1$$

$$= \boxed{\left(-\frac{5-2t}{s}e^{-st} + \frac{2}{s^2}e^{-st}\right)\Big|_0^2} \quad \frac{1}{2}$$

$$+ \boxed{\lim_{N\to\infty}\left(-\frac{t-1}{s}e^{-st} - \frac{1}{s^2}e^{-st}\right)\Big|_2^N} \quad 1$$

$$= \boxed{\left(-\frac{1}{s}e^{-2s} + \frac{2}{s^2}e^{-2s} - \left(-\frac{5}{s} + \frac{2}{s^2}\right)\right)} \quad \frac{1}{2}$$

$$+ \boxed{\lim_{N\to\infty}\left(-\frac{N-1}{s}e^{-sN} - \frac{1}{s^2}e^{-sN} + \frac{1}{s}e^{-2s} + \frac{1}{s^2}e^{-2s}\right)} \quad \frac{1}{2}$$

$$= -\frac{1}{s}e^{-2s} + \frac{2}{s^2}e^{-2s} + \frac{5}{s} - \frac{2}{s^2} + \boxed{-0-0+\frac{1}{s}e^{-2s} + \frac{1}{s^2}e^{-2s}} \quad \frac{1}{2}$$

OK IF "−0−0" NOT WRITTEN

$$= \boxed{\frac{5}{s} - \frac{2}{s^2} + \frac{3}{s^2}e^{-2s}} \quad \frac{1}{2}$$

$$\lim_{N\to\infty}\frac{N-1}{s}e^{-sN}$$

$$= \lim_{N\to\infty}\frac{N-1}{se^{sN}}$$

$$= \boxed{\lim_{N\to\infty}\frac{1}{s^2e^{sN}}} = 0 \quad \frac{1}{2}$$

$(s>0)$

[4] $f(t) = \begin{cases} 2t+1, & 0<t<1 \\ 2, & t>1 \end{cases} = 2t+1+u(t-1)(2-(2t+1))$ ½

$= 2t+1+u(t-1)(1-2t)$ ½

$\mathcal{L}\{f\} = \frac{2}{s^2}+\frac{1}{s}+e^{-s}\mathcal{L}\{1-2(t+1)\}$

$= \frac{2}{s^2}+\frac{1}{s}+e^{-s}\mathcal{L}\{-2t-1\}$ ½

½ $= \frac{2}{s^2}+\frac{1}{s} - e^{-s}\left(\frac{2}{s^2}+\frac{1}{s}\right)$ ½

½ $s^2Y - s\overset{-3}{y(0)} - \overset{7}{y'(0)} + 4(sY - \overset{-3}{y(0)}) + 4Y = \frac{2}{s^2}+\frac{1}{s}-e^{-s}\left(\frac{2}{s^2}+\frac{1}{s}\right)$

$s^2Y+3s-7+4sY+12+4Y = \frac{2}{s^2}+\frac{1}{s}-e^{-s}\left(\frac{2}{s^2}+\frac{1}{s}\right)$

½ $(s^2+4s+4)Y = -3s-5+\frac{2}{s^2}+\frac{1}{s}-e^{-s}\left(\frac{2}{s^2}+\frac{1}{s}\right)$

$Y =$ ½ $\frac{-3s-5}{(s+2)^2}+\frac{2+s}{s^2(s+2)^2}-e^{-s}\left(\frac{2+s}{s^2(s+2)^2}\right)$

$= \frac{-3(s+2)+1}{(s+2)^2}+\frac{1}{s^2(s+2)}-e^{-s}\left(\frac{1}{s^2(s+2)}\right)$ ½

1 $= -\frac{3}{s+2}+\frac{1}{(s+2)^2} - \frac{\frac{1}{4}}{s}+\frac{\frac{1}{2}}{s^2}+\frac{\frac{1}{4}}{s+2}$ 1½ $- e^{-s}\left(\frac{-\frac{1}{4}}{s}+\frac{\frac{1}{2}}{s^2}+\frac{\frac{1}{4}}{s+2}\right)$

$y =$ ½ $-3e^{-2t}+te^{-2t} - \frac{1}{4}+\frac{1}{2}t+\frac{1}{4}e^{-2t}$ ½ $- u(t-1)\left(-\frac{1}{4}+\frac{1}{2}(t-1)+\frac{1}{4}e^{-2(t-1)}\right)$ 1

½ $= -\frac{11}{4}e^{-2t}+te^{-2t}-\frac{1}{4}+\frac{1}{2}t-u(t-1)\left(-\frac{3}{4}+\frac{1}{2}t+\frac{1}{4}e^{-2t+2}\right)$

$\frac{1}{s^2(s+2)} = \frac{A}{s}+\frac{B}{s^2}+\frac{C}{s+2}$ ½ $\longrightarrow 1 = As(s+2)+B(s+2)+Cs^2$ ½

SANITY CHECK $s=2$: $\frac{1}{4(4)} = \frac{1}{16}$

$\frac{-\frac{1}{4}}{2}+\frac{\frac{1}{2}}{4}+\frac{\frac{1}{4}}{4} = -\frac{1}{8}+\frac{1}{8}+\frac{1}{16}$ ½

$= \frac{1}{16}$ ✓

$s=0$: $1 = 2B \rightarrow B = \frac{1}{2}$

$s=-2$: $1 = 4C \rightarrow C = \frac{1}{4}$

COEF OF s^2: $0 = A+C \rightarrow A = -C = -\frac{1}{4}$